Hot Spot Stability Analysis available in MADYN 2000 Version 4.2

Friction on the shaft surface can cause vibration-driven heat input to the shaft and thus hot spots. The resulting thermal bending can lead to unstable vibrations under certain circumstances (see for example /1/, /2/ and /3/).

The hot spot stability analysis is the main new feature in version 4.2 of MADYN 2000. It is offered as a new module. The implemented method (see among others /1/) has been used by DELTA JS for almost 20 years, by means of special in-house software, which was combined with MADYN classic. A lot of experience in numerous projects has been gained over the years and the method has been continuously refined.

The module offers the utmost flexibility. It allows handling practically any hot spot mechanism driven by vibration.

For hot spots caused in fluid film bearings (Morton Effect) the differential heating is estimated by methods based on the bearing power losses, which can be calculated by different methods including the ALP3T CFD analysis. In a further step the implementation of a specialized CFD code considering the exact perturbation due to any orbit and including the numerical analysis of the differential heat dissipation is planned (see /4/). This method is currently validated against DELTA JS' vast experience.

Moreover, past experience has shown, that hot spots cannot be considered isolated, because they influence each other. For example in case of a shaft with two overhangs and two bearings with potential Morton effect, the hot spot in both bearings must be considered at the same time. Otherwise the stability thresholds are not accurate. DELTA JS has analysed examples with 38 hot spots considered simultaneously in one model. The hot spots in that case were caused by brush seals.

The whole process of assessing the hot spot sensitivity of a rotor system in MADYN 2000 is described in the following.

Table of content for detailed description of the hot spot stability analysis:

References:

/1/ Joachim Schmied, Josef Pozivil, Joseph Walch: Hot Spots in Turboexpander Bearings: Case History, Stability Analysis, Measurements and Operational Experience. Proceedings of the ASME IGTI Conference, Berlin, 2008.

/2/ R. Gasch, R. Nordmann, H. Pfützner: Rotordynamik, 2. Auflage. Springer Verlag, 2002.

/3/ Lothar Eckert, Joachim Schmied, Alfred Ziegler: Case History and Analysis of the Spiral Vibration of a Large Turbogenerator Using Three Different Types of Heat Input. Proceedings of the 7th IFToMM International Conference on Rotordynamics, September 25-28 2006, Vienna Austria.

/4/ B.S. Grigoriev, A.E. Fedorov, Joachim Schmied: New Mathematical Model for the Morton Effect Based on the THD Analysis. Proceedings of the 9th IFToMM International Conference on Rotordynamics, 2014, Milan Italy.

1. Modelling a Hot Spot HSP

Hot spots can arise in fluid film bearings (in this case the phenomenon is called Morton effect), at slip rings of electric machines, at brush seals and due to light contact. Other effects, such as eddy current losses in magnetic bearings, can lead to hot spots as well.

The model of the hot spot is based on Kellenberger. It requires the definition of a heat input factor p and a heat dissipation factor q (see /1/, and references there).

Parameters of a hot spot are defined in a special object "Hot Spots" HSP. The objects are added to the shaft stations, where the hot spots arise. A station can have one hot spot, thus shafts and systems several hot spots.

Two types of hot spots can be defined:

- User-Defined
- Morton Effect

For user defined hot spots, all parameters must be defined by the user. This allows modelling a large variety of causes for hot spots (friction at slip rings, brush seals …).

The Morton Effect type is limited to hot spots caused by the friction in fluid film bearings. It requires that a fluid film bearing RFB is at the same station. All parameters for the Morton Effect type are automatically determined, except the angle Φ_T (see following sub-chapters 1.1 and 1.2).

It should be noted that the Morton Effect can also be modelled as User-Defined hot spot.

In the following a model with hot spots in both fluid film bearings is shown. Note the new symbol ⚡ for hot spots.
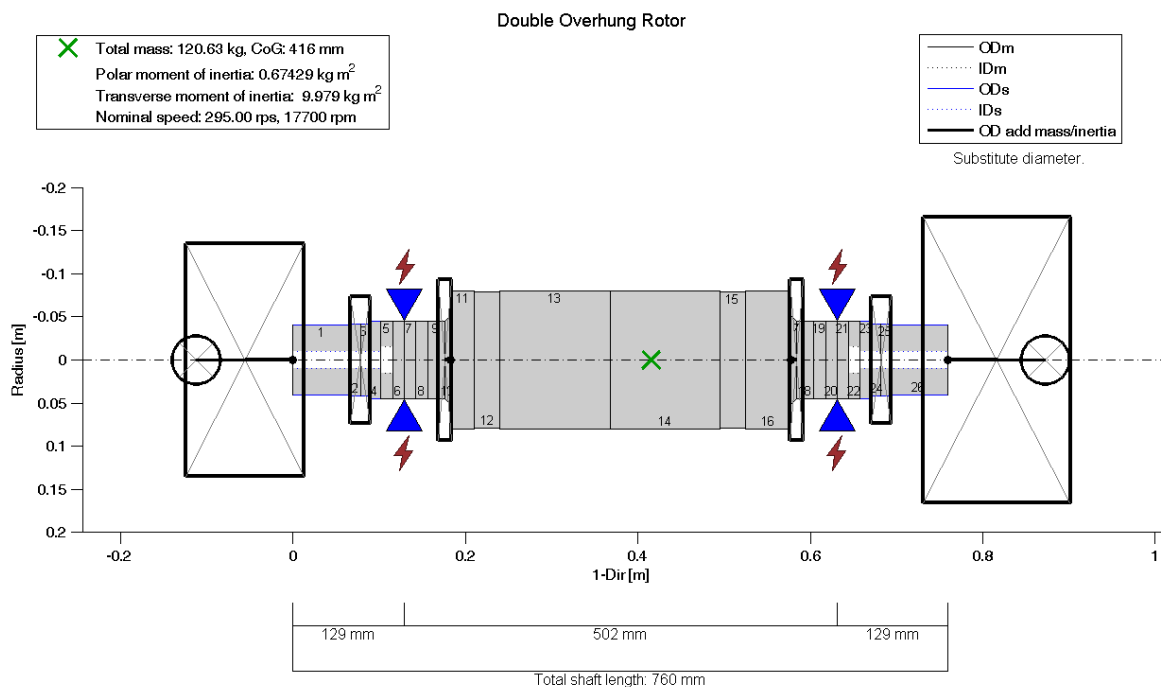
Fig. 1.1: Example with two bearing hot spots

1.1 User Defined Hot Spot

A hot spot HSP is defined by the following parameters:

1. Axial length L_T of the area with cross-sectional temperature difference.
2. In case of superimposed sections one section subject to the temperature difference must be selected.
3. Type of heating: The heat input can be proportional to the acceleration, velocity or displacement of a station, i.e. $\sim\ddot{x}$, $\sim\dot{x}$, $\sim x$.
4. Proportionality factor p for the heat input. This factor can be a constant or a function of speed. The input must be defined for β=1m/K, with β as the ratio between cross sectional temperature difference and the thermal deflection at a reference point. MADYN 2000 automatically chooses the hot spot location as the reference point (also see chapter 2).
5. Proportionality factor q for the eliminated heat. This factor can be a constant or a function of speed.
6. Angle Φ_T between high spot (direction of acceleration, velocity or displacement) and maximum of cross sectional temperature difference (see figure 1.2). Either a range of speed-independent angles or one speed dependent angle can be defined.

The GUI to define the parameters can be seen in figure 1.3.

It is assumed, that the cross sectional temperature difference along the length L_T is constant. More than one section on each side of the hot spot station can be subject to the temperature difference, depending on L_T (see figure 1.4).

In case sections are only partially affected, the load at the left and right station of the section due to the temperature difference is multiplied by a factor $\lambda = \frac{L_{Section\ affected\ by\ Hot\ Spot}}{L_{Section}}$.

The definition of p and q is not obvious, but is kept deliberately open, in order to allow the utmost flexibility. In chapter 5 formulas for their definition are given for some cases.
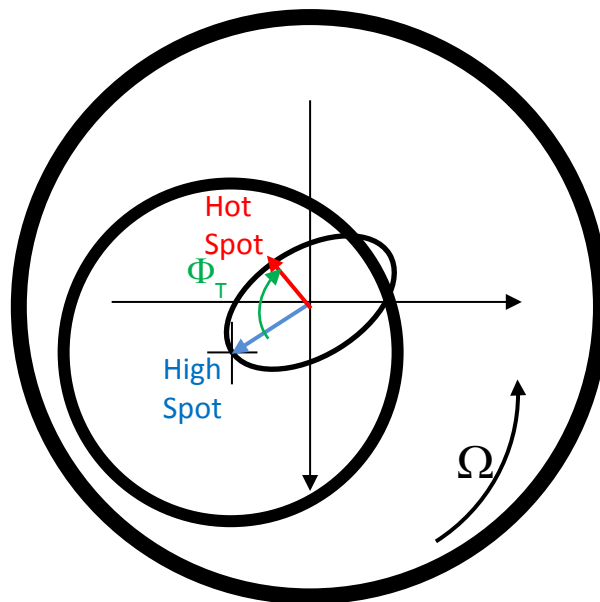

Fig. 1.2: Definition of Φ_T (The angle Φ_T between high spot and hot spot is averaged for one revolution of the shaft. Φ_T is positive in the direction of rotation. The angle in the figure is the angle for one instant.)
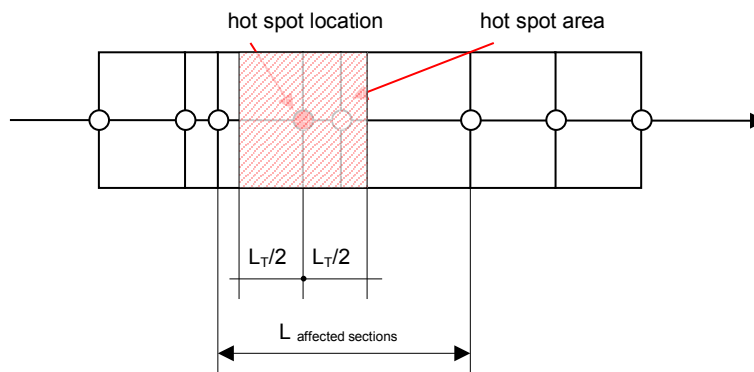
Fig.1.3: GUI to define a hot spot

Fig. 1.4: Affected sections of a hot spot

1.2 Morton Effect

For the "Morton Effect" analysis (hot spot caused by fluid film bearings) the heat input factor and heat dissipation factor are calculated automatically from the parameters of the bearing. Details of how they are determined are given in chapter 5.1.

The GUI to define a hot spot according to the Morton Effect is shown in figure 1.5.

The only parameter, which can be defined manually, is the hot spot angle Φ_T (see figure 1.2). It is known from measurements (see for example De Jongh and Moron referenced in /1/) that this angle can deviate from zero and tends to be negative. In order to see its influence a variation of angles can be input.
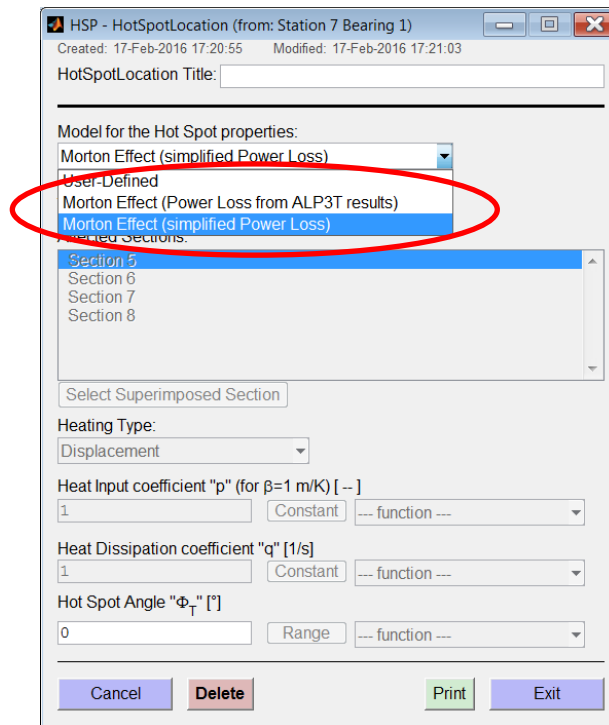
Fig.1.5: GUI to define a hot spot according to the Morton Effect

The differential heat input is calculated from the bearing power loss. For this calculation two options are offered:

1.) Simplified Power Loss: The power loss is calculated analytically from a simple formula applicable to a rotor centred in a cylindrical bearing with constant viscosity.
2.) Power Loss from ALP3T results: The power loss is calculated from a CFD analysis (ALP3T) considering the eccentricity of the journal in the bearing due to load and the accurate bearing geometry. The temperature distribution and variable viscosity can be considered as well depending on the selected type of analysis for the bearing characteristics in the RFB object.

Estimation of the factors according to option 1 (simplified Power Loss):

The assumptions for this option are summarized in chapter 5.1. The analysis gives a correct order of magnitude for the heat input and the heat dissipation.

The required parameters are:

Bearing width B
Bearing diameter D
Rotor speed Ω
Oil viscosity according to the mean temperature η
Radial bearing clearance Δr
Density of the affected shaft sections ρ
Specific heat capacity of the affected shaft sections c
Heat transfer coefficient between oil and rotor $\alpha = \frac{2\,Nu\,\lambda_{Oil}}{\Delta r}$, which requires
Nusselt number $Nu = 2.6$
Thermal conductivity of the oil λ_{Oil}

With these parameters the factor q and the heat ratio $\frac{p\Omega}{q}$ ($\rightarrow$ factor p) can be calculated as follows (also see /1/ and chapter 5.1):

$$q = \frac{3\,\alpha\,A}{m\,c} \tag{1.1}$$

$$\frac{p\Omega}{q} = \frac{u^2\,\eta\,\beta}{\Delta r^2\,\alpha} \tag{1.2}$$

with $A = \pi\,D\,B$ as the shaft surface of the bearing and $u = \frac{D}{2}\,\Omega$ as the circumferential shaft surface speed.

The axial length L_T of the area with cross-sectional temperature difference (fig. 1.4) is set equal to the bearing width.

Estimation of the factors according to option 2 (accurate Power Loss from ALP3T results):

In this option an accurate power loss calculated by the ALP3T CFD analysis is used instead of a simplified formula (see chapter 5.1). This results in the following formula for the heat ratio:

$$\frac{p\Omega}{q} = \frac{P\beta}{\Delta r\,\alpha} \quad , \tag{1.3}$$

with P as the bearing power loss from ALP3T.

It should be noted here, that this method leads to an overestimation of the heat input for the hot spot in case of turbulence in a bearing, which is considered in a variable adiabatic ALP3T analysis.

Specialized CFD analysis (planned for near future)

The analysis with estimated factors can be used to assess the stability with respect to p and q, i.e. it can be analysed at which heat ratio $\frac{p\Omega}{q}$ or at which heat input factor p the system becomes unstable (also see /1/, chapter 3 and 4).

The specialized CFD analysis then can be used in a further step to calculate the actual p and q factors and the actual heat ratio $\frac{p\Omega}{q}$, respectively (see /4/). It also provides the hot spot angle Φ_T. This analysis considers the accurate bearing geometry, the static journal position in the bearing, the accurate perturbation of the power losses by the orbit for the differential heat input and the numerically calculated heat dissipation (instead its analysis by a heat transfer coefficient). The method is more accurate than option 2 regarding the perturbation by the orbit and the heat dissipation.

The method is applied only to bearings, which were calculated according to DIN (ALP3T_DinTab) or with a variable adiabatic analysis (ALP3T_v_ad).

The following parameters and assumptions are used:

    Constant viscosity according to the mean temperature in case of DIN
    Variable viscosity according to the 3-dimensional temperature distribution in case of DIN and v_ad
    Density of the oil according to mean (for DIN) or inlet temperature (for v_ad)
    Heat capacity of the oil according to mean (for DIN) or inlet temperature (for v_ad)
    Thermal conductivity of the oil according to mean (for DIN) or inlet temperature (for v_ad)
    Density of the affected shaft sections ρ
    Specific heat capacity of the affected shaft sections *c*
    Heat conductivity of the shaft 51 W/(m K)
    Ambient temperature 20°C (for heat dissipation through shaft)
    Heat transfer coefficient for heat dissipation through shaft 115 W/(m^2 K)
    No heat dissipation through the pads
    Orbit of the shaft at the hot spot location
    Static position of the journal at the hot spot location

2. Thermal Deformation Analysis

The "Thermal Deformations Analysis" is required for a hot spot stability analysis (see chapter 3).

The analysis requires that any shaft in the system has defined hot spots. No loads must be defined. The hot spots in the system are selectable for the analysis. The GUI to define the analysis can be seen in the following figure.
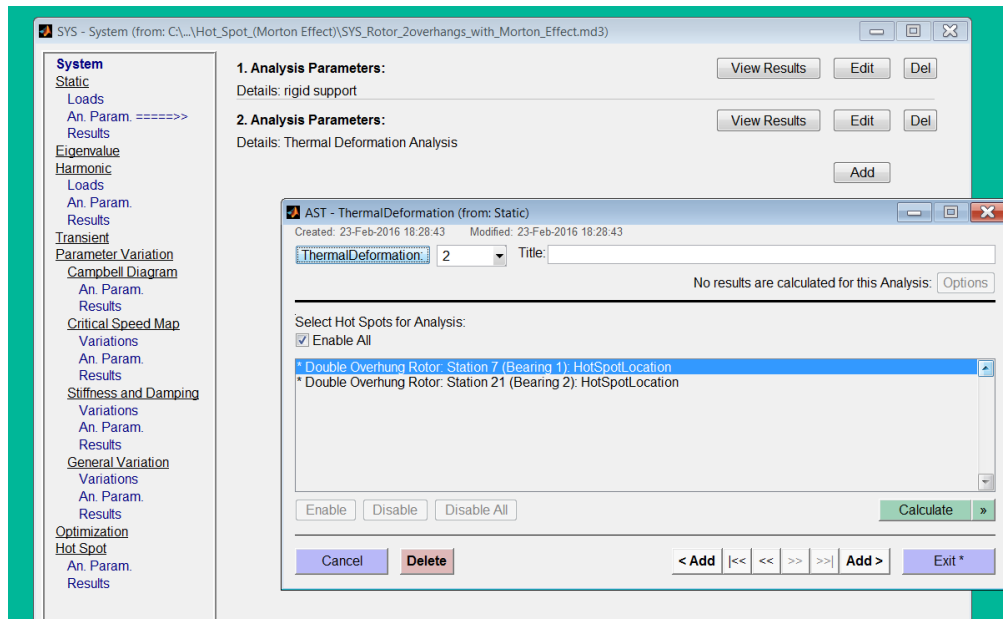

Fig. 2.1: GUI to define the thermal deformation analysis (called from "Static An. Param." pane)

The purpose of the analysis is to calculate the thermal deformation for 1°C cross sectional temperature difference in the area of each selected hot spot (along the length L$_T$, see figure 1.4). The analysis yields the parameter β (see chapter 1), which is the thermal deformation at a reference point (the hot spot location) per cross sectional temperature difference.

The deformation shall apply for the free shaft. Since a free shaft does not allow a static analysis, because of the singularity of the stiffness matrix, some boundary conditions have to be applied, which do not cause any constraint forces, i.e. each shaft must be statically determined (not overdetermined). This is done automatically, by clamping each affected shaft without bearings and supports at a suited station.

The thermal deformation for a hot spot in bearing 2 of the example in figure 1.1 is shown in figure 2.2.
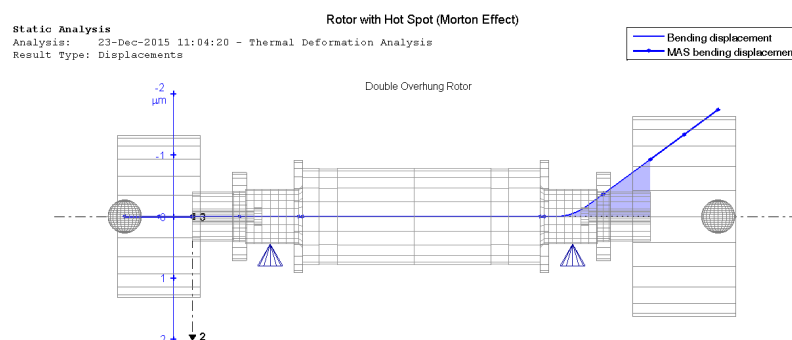

Fig.2.2: Thermal deformation for a hot spot in bearing 2 (example in figure 1.1)

3. Hot Spot Stability Analysis

The hot spot stability analysis can be carried out, if a system contains one or several hot spots. It is a similar analysis to a Campbell diagram, i.e. it calculates eigenvalues and natural modes as a function of speed. The system, which is solved for eigenvalues, is extended by the thermal equations for the hot spots (see /1/). The results consisting of the complete set of eigenvalues are filtered to find only the eigenvalues, which are caused by the hot spot. They are characterized, by their frequency, which is close to the rotor speed, and their damping ratio, which is low or negative, since their real part is in the order of magnitude of –q (with q as the factor for heat dissipation).

The filtered hot spot eigenvalues and mode shapes represent the response of the system to the thermal bending of the shaft. A positive real part of an eigenvalue ($\rightarrow$ negative damping ratio) means that the response is unstable.

The GUI to define the parameters for the hot spot analysis can be seen in figure 3.1.
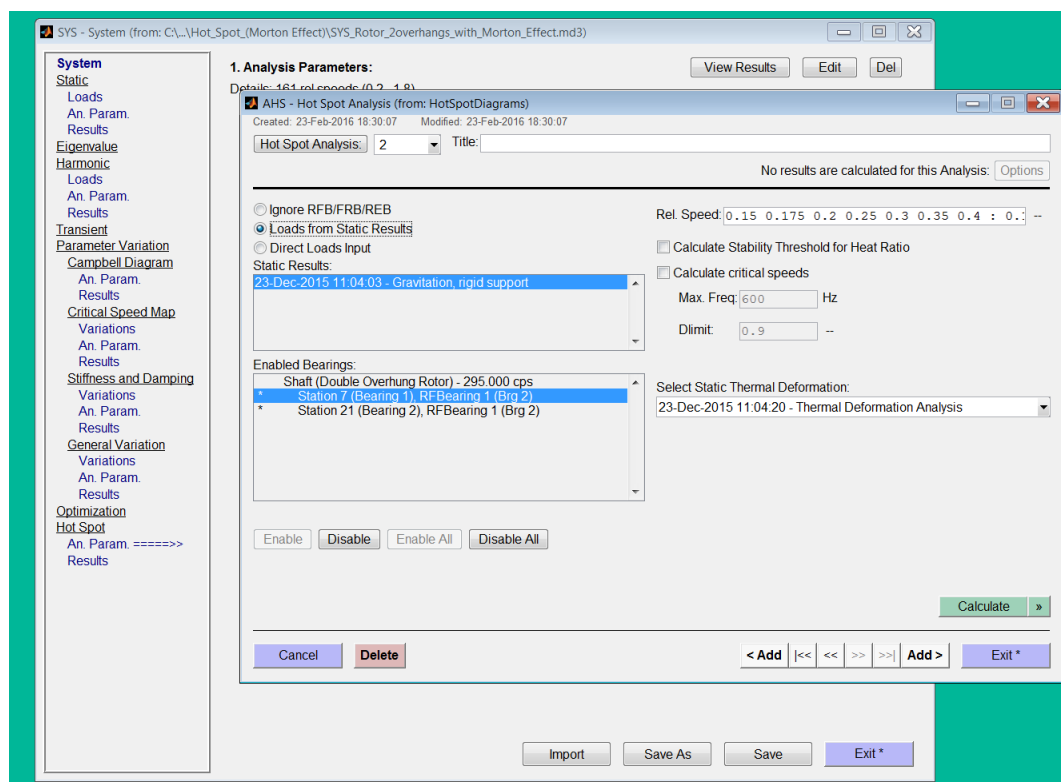
Fig. 3.1 GUI for the hot spot analysis

Most of the hot spot analysis parameters are similar to a Campbell diagram. They are:

1. The static load of RFBs, FRBs and REBs.
2. In case RFBs with matrices for non-synchronous characteristics to determine transfer functions TFU are present, the button "as TFU" is active. It allows specifying the use of transfer functions instead of the synchronous stiffness and damping. Bearings, for which transfer functions are used, are marked with (T) in the list of RFBs.
3. In case dynamic bearing supports DBS are present a check box allows to consider them, i.e. all SBS for which DBS have been defined are substituted. The check box can only be activated if polynomials of the transfer functions were created or state space matrices are present.

4. A check box allows the determination of the critical speeds from the complete set of eigenvalues before filtering the hot spot eigenvalues. They can be used in the result plots to assess the hot spot stability (see chapter 4). In this case a frequency range and damping ratio can be defined for filtering the modes to be considered.
5. Another check box "Calculate Stability Threshold for Heat Ratio" checks, if all hot spots in the system are of the same type and have the same p (for the actual β) and q values for the added and dissipated heat, since some result plots are only available, if this condition is fulfilled (also see next chapter).
6. The static analysis with the thermal deformations.

In case all hot spots are of the same type and have same p (for the actual β) and q factors the threshold for the stability of each hot spot can be calculated by a common dimensionless heat ratio $\frac{p\Omega}{q}$ (for displacement type), $\frac{p\Omega^2}{q}$ (for velocity type) or $\frac{p\Omega^3}{q}$ (for acceleration type). The threshold is extrapolated from the real part of the hot spot eigenvalue and real part for p=0, which is –q.

The equality of p and q is checked with a tolerance of 10%. If the values are equal within this tolerance, they are treated as equal and the mean value is used for the heat ratio.

In case the hot spots are different a second eigenvalue analysis is carried out with all p values multiplied by 1.1. The two sets of results allow calculating thresholds for each hot spot mode.

4. Plots to Assess the Hot Spot Stability

The GUI to call result plots for the assessment of the hot spot sensitivity of a system is shown in figure 4.1. There are plots for the stability threshold and plots directly based on the eigenvalues.
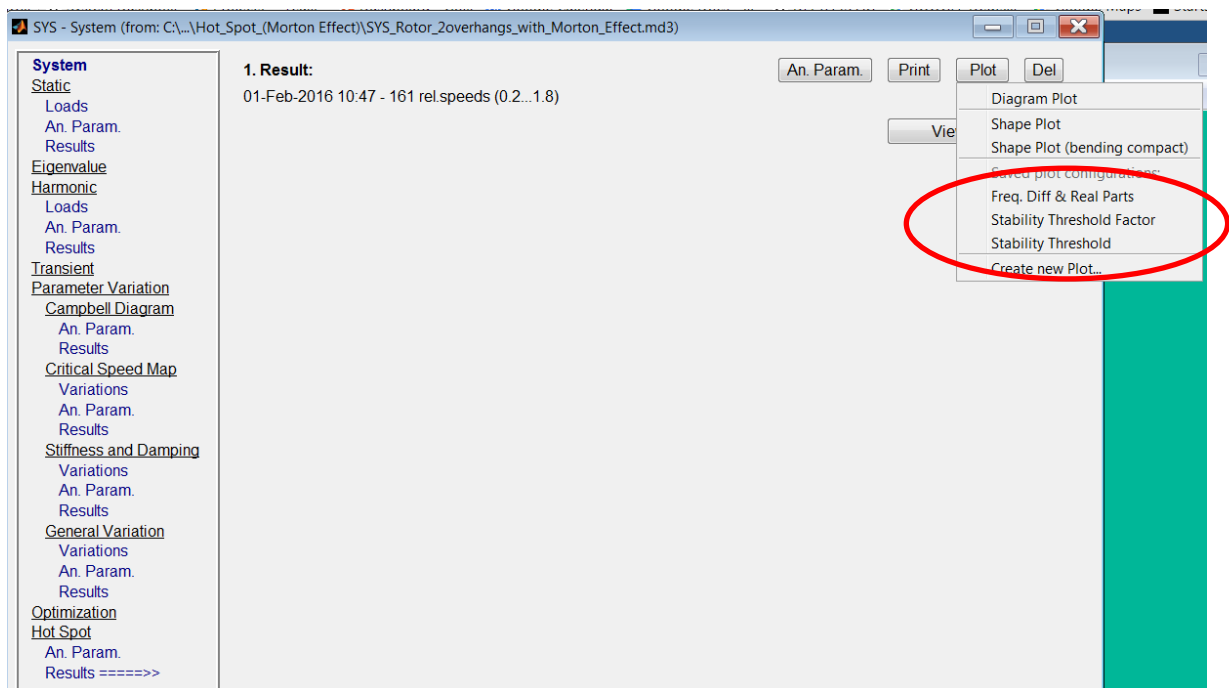

Fig. 4.1: GUI to call plots for results of a hot spot analysis

4.1 Plots for the Stability Threshold

The following diagrams are offered for the stability threshold:

- Plot for the "Stability Threshold Factor"
- Plot for the "Stability Threshold for the Heat Ratio"

The plot for the **"Stability Threshold Factor"** can always be applied, even for different hot spot types and p / q factors for the heat input / dissipation. It is the factor by which existing p values must be multiplied in order to push the most unstable eigenvalue (eigenvalue with the larges real part) to the stability threshold (real part of the eigenvalue zero). A factor smaller than 1 means, that the system with the existing p and q values is unstable, whereas a factor larger than 1 means, it is stable. The threshold is at a factor of 1.

An example of such a plot for the system in figure 1.1 is shown in figure 4.2. The example has 2 hot spots and the lowest threshold factor is determined by either of the 2 resulting hot spot modes. The stability threshold factor is below 100% for speeds higher than 16'500rpm. The system then is unstable.

The plot for the **"Stability Threshold for the Heat Ratio"** requires equal p and q factors (within a tolerance of 10%) for the added and dissipated heat of all hot spots in the system. The stability threshold of all hot spot modes then can be plotted as a common dimensionless heat ratio $\frac{p\Omega}{q}$ (for displacement type), $\frac{p\Omega^2}{q}$ (for velocity type) or $\frac{p\Omega^3}{q}$ (for acceleration type). The actual heat ratio can also be selected to be shown in the same plot. This heat ratio is either calculated from the input p and q values, or in case of the Morton effect from equation 1.2 or 1.3 depending on the used option for the heat input calculation. Additionally, for the Morton effect an even more accurate ratio calculated with a specialized CFD analysis (see /4/) can be shown (planned for the near future).

In figure 4.3 a plot with the stability threshold for the heat ratio is shown together with the actual heat ratio for the same example as in figure 4.2. It can be seen, that hot spot mode 2 has a lower threshold up to 29'000rpm and that the actual heat ratio is higher than the threshold for speeds higher than 16'500rpm, which is in agreement with the threshold speed in figure 4.2.

The shapes associated with the 2 hot spot modes at nominal speed can be seen in figure 4.4. They represent the response to the thermally bent shaft relative to the unbent shaft.

4.2 Plots directly Derived from Eigenvalues

Plots derived from the filtered hot spot eigenvalues are:

- Hot Spot Frequency Difference
- Real Part of Eigenvalue

These plots are available for any p and q factors. The plot with the frequency difference and the real part of the eigenvalue for the same example as in the previous figures is shown in figure 4.5.

The **"Hot Spot Frequency Difference"** is the difference to the rotor speed. It corresponds to the frequency the hot spot is moving on the rotor. In case of a positive frequency difference the hot spot is moving in the same direction as the speed and in case of a negative difference against the rotor speed. A phase angle change in a measurement will be accordingly. The frequency difference corresponds to the frequency of the rotating phase angle, which can normally be observed when measuring the phenomenon.

The **"Real Part of Eigenvalue"** tells, if the system is stable or unstable with the p and q factors for the heat input and the heat dissipation as they were used in the analysis. For a positive real part the system is unstable.

In case all hot spots have a common q factor, this factor can be included in the plot. In case any real part is smaller than –q it means, that the hot spot cannot lead to instability even for very large heat input (large p factor). At speeds larger than 29'000rpm this is the case for the real part of hot spot 2. In the plot for the stability threshold for the heat ratio the threshold is infinite at these speeds.
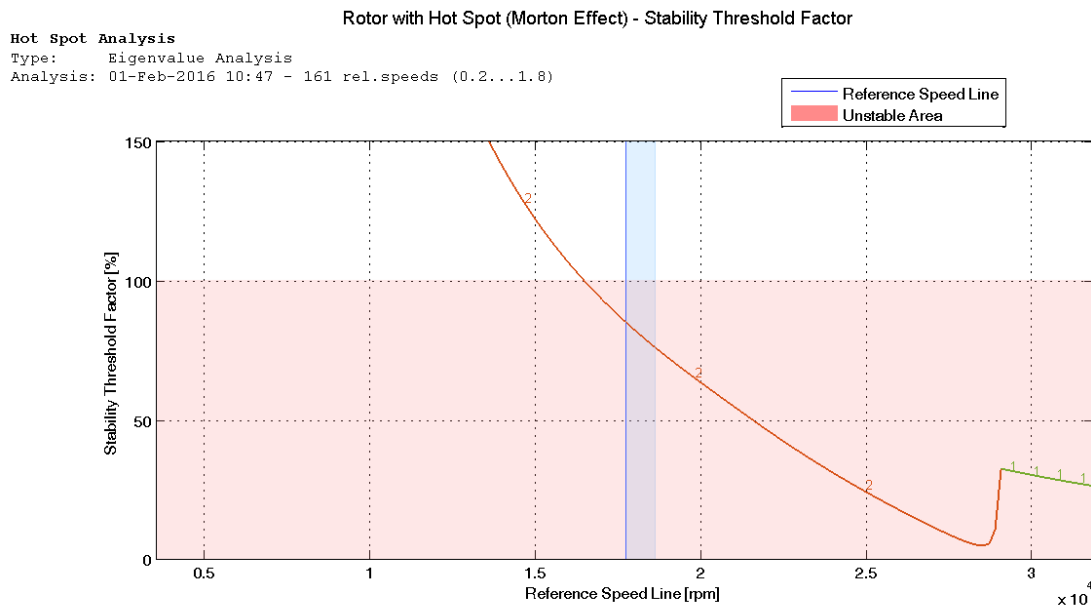

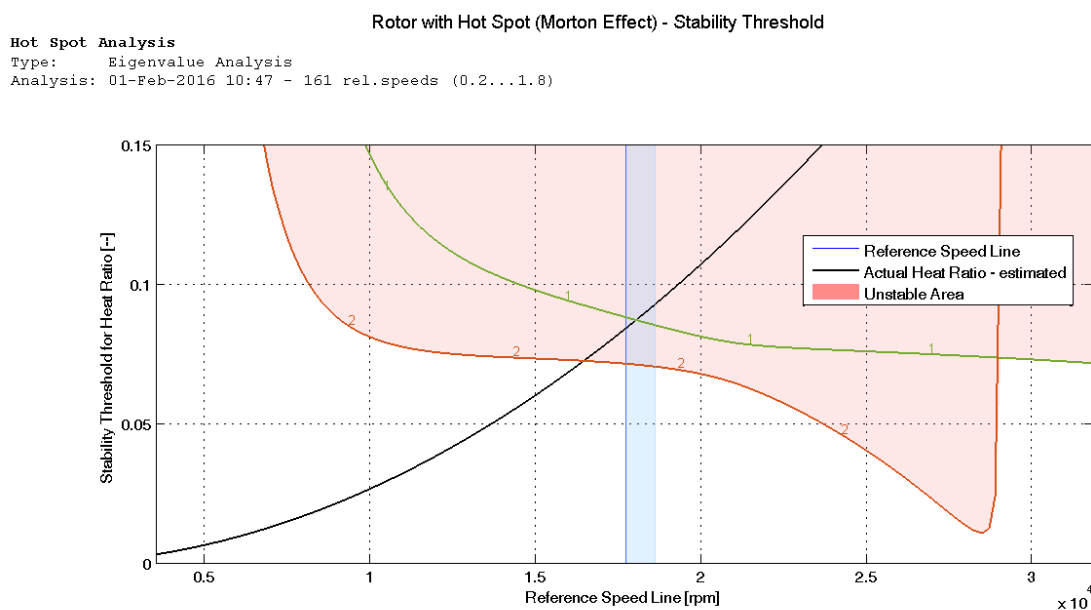Fig. 4.2: Plot for the stability threshold factor

Fig. 4.3: Plot for the stability threshold of the heat ratio $\frac{p\Omega}{q}$

Rotor with Hot Spot (Morton Effect)

```
Parameter Variation Analysis
Type:      Parameter Variation
Analysis: 01-Feb-2016 10:47 - 161 rel.speeds (0.2...1.8)

Shape for:  17700.00 rpm
Mode:       1
Frequency:  295.01 Hz
            17700.3 cpm
Damping:    0.0 %

Whirling direction: +0.99
```

Bending displacement
MAS bending displacement

Double Overhung Rotor

MADYN 2000 v.4.2.development

Rotor with Hot Spot (Morton Effect)

```
Parameter Variation Analysis
Type:      Parameter Variation
Analysis: 01-Feb-2016 10:47 - 161 rel.speeds (0.2...1.8)

Shape for:  17700.00 rpm
Mode:       2
Frequency:  295.00 Hz
            17700.2 cpm
Damping:    -0.0 %

Whirling direction: +0.98
```

Bending displacement
MAS bending displacement

Double Overhung Rotor

MADYN 2000 v.4.2.development

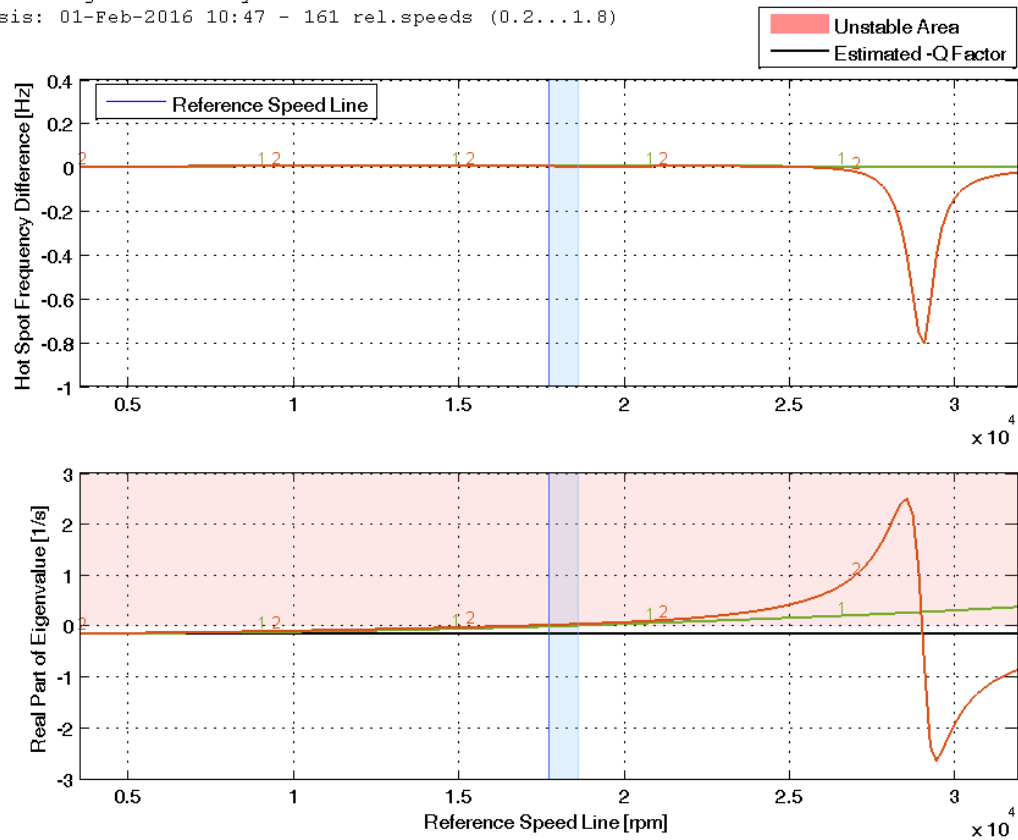Fig. 4.4: Shapes of the hot spot modes at nominal speed 17'700rpm

Fig. 4.5: Plot with frequency difference (to speed), the real part and the common q factor

5. Heat Input and Heat Dissipation Factors for the Hot Spot Stability Analysis

5.1 Hot Spot Caused by Fluid Film Bearings (Morton Effect)

Analysis with the heat input from a simplified analytical formula for the bearing power loss

The simplifying assumptions for this estimation of the p, q factor are as follows:

The journal orbit is a synchronous circle.
The radial gap around the circumference is constant.
The circumferential distribution of the added heat due to the vibration is a sinus shape. The maximum is at the high spot of the shaft.
For the eliminated heat distribution the same applies as for the added heat.
The axial distribution of the temperature in the journal is constant.
The journal ends are adiabatic.
The complete sinus shaped friction power due to the vibration enters the shaft (not the oil).

Denotations:

A   surface area of the journal surrounded by oil
Δr radial bearing clearance
c   specific heat capacity of the shaft material
m   mass of the journal
p   proportionality factor for the added heat
P   power loss in the bearing
q   proportionality factor for the eliminated heat

$\Delta \dot{Q}$ added heat due to the vibration

u   circumferential speed of the shaft
x   radius of the circular shaft orbit
x_T thermal deflection

Θ cross sectional temperature difference
α heat transfer coefficient in the bearing
β ratio of the thermal deflection in the bearing to the cross sectional temperature difference
radial bearing clearance
η oil viscosity
Ω shaft speed in rad/s

Thermal equation for the cross sectional temperature difference Θ in the journal:

$$\dot{\Theta} + \kappa \Theta = K \tag{5.1.1}$$

with

$$\kappa = \frac{3\alpha A}{mc}, \tag{5.1.2}$$

$$K = \frac{3\pi}{2mc} \Delta \dot{Q} \tag{5.1.3}$$

Thermal equation for the thermal bending of the journal according to Kellenberger :

$$\dot{x}_T = p\Omega x - qx_T \tag{5.1.4}$$

The relation between x_T and θ is:

$$x_T = \beta\Theta \tag{5.1.5}$$

(5.1.5) substituted into (5.1.4) yields:

$$\dot{\Theta} + q\Theta = \frac{p\Omega}{\beta}x \tag{5.1.6}$$

Comparing (5.1.6) and (5.1.1) considering (5.1.2) and (5.1.3) yields:

$$\boxed{q = \frac{3\alpha A}{mc}}, \tag{5.1.7}$$

$$\frac{p\Omega}{\beta}x = \frac{3\pi}{2mc}\Delta\dot{Q} \tag{5.1.8}$$

(5.1.7) and (5.1.8) yield:

$$\frac{p\Omega}{q} = \frac{\pi\Delta\dot{Q}\beta}{2x\alpha A} \tag{5.1.9}$$

For a bearing and heating with the mentioned simplifications the added heat can be estimated as follows:

$$\Delta\dot{Q} = \frac{2}{\pi}P\frac{x}{\Delta r}, \tag{5.1.10}$$

with

$$P \approx \frac{u^2\eta A}{\Delta r} \tag{5.1.11}$$

Equation (5.1.10) hereby represents the perturbation of the power losses by the vibration on a circular orbit. In equation (5.1.11) the power loss of a cylindrical unloaded bearing is used as an estimate of the real power loss.

(5.1.10) and (5.1.11) substituted into (5.1.9) yields:

$$\boxed{\frac{p\Omega}{q} = \frac{u^2\eta\beta}{\Delta r^2\alpha}} \tag{5.1.12}$$

<u>Analysis with the heat input from the bearing power loss calculated accurately by a CFD analysis (ALP3T)</u>

In equation (5.1.10) we can use the accurately calculated power loss instead the power loss estimated from an unloaded cylindrical bearing according to equation (5.1.11). This power loss considers the real bearing load and the real bearing geometry. Substituting (5.1.10) directly into (5.1.9) yields

$$\boxed{\frac{p\Omega}{q} = \frac{P\beta}{\Delta r \alpha A}} \tag{5.1.13}$$

The power loss now is accurately calculated, but the perturbation of the power loss represented by the factor of $\frac{2\,x}{\pi\,\Delta r}$ in equation (5.1.9) is still an approximation. This factor assumes a uniform perturbation around the circumference, which does not apply for the most bearings. Nevertheless it is a very good approximation, as experience from numerous projects proves.

5.2 Hot Spot Caused by Generator Slip Rings

In the following the estimation of the factors for the added and dissipated heat for a generator slip ring is described (see /3/).

Assumptions for the estimation:

The slip ring orbit is a synchronous circle.
The circumferential distribution of the added heat due to the vibration is a sinus shape. The maximum is at the high spot of the shaft.
For the eliminated heat distribution the same applies as for the added heat.
The axial distribution of the temperature in the slip ring is constant.
The slip ring is adiabatic.
The complete sinus shaped friction power due to the vibration enters the ring.
The thermal bending of the slip ring is completely transmitted to the shaft.

Denotations:

R_a outer radius of the slip ring
R_i inner radius of the slip ring
γ R_i/R_a
B width of the slip ring seat
c specific heat capacity of the shaft material
ρ density of the slip ring material
p proportionality factor for the added heat
q proportionality factor for the eliminated heat
F contact force of brushes
μ_s friction coefficient between brush and shaft
μ_h friction coefficient between brush and holder
u circumferential speed of the shaft
Ω shaft speed in rad/s

$\Delta \dot{Q}$ added heat due to the vibration

α heat transfer coefficient of the slip ring to ambience

x radius of the circular shaft orbit
x_T thermal deflection
β ratio of the thermal deflection at the slip ring to the cross sectional temperature difference

Thermal equation for the cross sectional temperature difference Θ in the journal:

$$\dot{\Theta} + \kappa \Theta = \mathrm{K} \tag{5.2.1}$$

with

$$\kappa = \frac{6\alpha}{\rho R_a c (1 - \gamma^3)}, \tag{5.2.2}$$

$$\mathrm{K} = \frac{3}{2\rho B R_a{}^2 c (1 - \gamma^3)} \Delta \dot{Q} \tag{5.2.3}$$

Thermal equation for the thermal bending of the slip ring for the case of a heat input proportional to the vibration velocity (inspired by Kellenberger) :

$$\dot{x}_T = p\Omega\, \dot{x} - q x_T \tag{5.2.4}$$

The relation between x_T and θ is:

$$x_T = \beta\Theta \tag{5.2.5}$$

(5.2.5) substituted into (5.2.4) yields:

$$\dot{\Theta} + q\Theta = \frac{p\Omega}{\beta}\dot{x} \tag{5.2.6}$$

Comparing (5.2.6) and (5.2.1) considering (5.2.2) and (5.2.3) yields:

$$\boxed{q = \frac{6\alpha}{\rho R_a c(1-\gamma^3)}}, \tag{5.2.7}$$

$$\frac{p\Omega}{\beta}\dot{x} = \frac{3}{2\rho B R_a^2 c(1-\gamma^3)}\Delta\dot{Q} \tag{5.2.8}$$

(5.2.7) and (5.2.8) yield:

$$\frac{p\Omega}{q} = \frac{\Delta\dot{Q}\beta}{4 B R_a \alpha\, \dot{x}} \tag{5.2.9}$$

The heat input $\Delta\dot{Q}$ caused by the variation of the contact force due to the friction in the brush holder can be expressed as follows:

$$\Delta\dot{Q} = F\frac{\dot{x}}{\Omega\hat{x}}\mu_s^2\mu_h u \tag{5.2.10}$$

Substituting (5.2.10) into (5.2.9) yields

$$\boxed{\frac{p\Omega^2}{q} = \frac{F\mu_s^2\mu_h\beta\Omega}{4 B\alpha\,\hat{x}}} \tag{5.2.11}$$

5.3 Hot Spot Caused by Brush Seals

In the following the estimation of the factors for the added and dissipated heat for brush seals is described.

Assumptions for the estimation:

The circumferential distribution of the added heat due to the vibration is a sinus shape. The maximum is at the high spot of the shaft.
For the eliminated heat distribution the same applies as for the added heat.
The resulting temperature difference in the shaft is sinus shaped.
An effective axial length subject to a temperature difference is assumed. In this area the temperature difference is assumed as constant.

Denotations:

A   surface area along the length subject to a temperature difference
c   specific heat capacity of the shaft material
m   mass of the journal along the length subject to a temperature increase
p   proportionality factor for the added heat
P   friction power
q   proportionality factor for the eliminated heat

$\Delta\dot{Q}$ added heat due to the vibration
k   contact stiffness between brush and rotor

u   circumferential speed of the shaft
x   radius of the circular shaft orbit
x_T thermal deflection

Θ cross sectional temperature difference
α heat transfer coefficient of the rotor surface
β ratio of the thermal deflection to the cross sectional temperature difference
Ω shaft speed in rad/s
μ friction coefficient between brush and rotor

Thermal equation for the cross sectional temperature difference Θ :

$$\dot{\Theta} + \kappa\Theta = K \qquad\qquad (5.3.1)$$

with

$$\kappa = \frac{3\alpha A}{mc} \qquad\qquad (5.3.2)$$

$$K = \frac{3\pi}{2mc}\Delta\dot{Q} \qquad\qquad (5.3.3)$$

Thermal equation for the thermal bending of the journal according to Kellenberger :

$$\dot{x_T} = p\Omega x - qx_T \qquad\qquad (5.3.4)$$

The relation between x_T and θ is:

$$x_T = \beta\Theta \qquad\qquad (5.3.5)$$

(5.3.5) substituted into (5.3.4) yields:

$$\dot{\Theta} + q\Theta = \frac{p\Omega}{\beta}x \tag{5.3.6}$$

Comparing (5.3.6) and (5.3.1) considering (5.3.2) and (5.3.3) yields:

$$\boxed{q = \frac{3\alpha A}{mc}} \tag{5.3.7}$$

$$\frac{p\Omega}{\beta}x = \frac{3\pi}{2mc}\Delta\dot{Q} \tag{5.3.8}$$

(5.3.7) and (5.3.8) yield:

$$\frac{p\Omega}{q} = \frac{\pi\,\Delta\dot{Q}\,\beta}{2\,x\,\alpha A} \tag{5.3.9}$$

The added heat can be calculated as follows:

$$\Delta\dot{Q} = k\,x\,\mu\,u \tag{5.3.10}$$

Substituting (5.3.10) into (5.3.9) yields the ratio of added to eliminated heat:

$$\boxed{\frac{p\,\Omega}{q} = \frac{\pi\,k\,\mu\,u\,\beta}{2\,\alpha A}} \tag{5.3.11}$$