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Analysis of Systems with Complex Gears

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Abstract

The requirements for the modelling of complex gears are explained. This includes model checks to guide the user. Models and analyses are demonstrated for three examples. For a model of a compressor train with a parallel-shaft gear considering coupling of lateral and torsional vibrations the contribution of the gear bearings to damping and stability is shown. For a train with a multi-stage planetary gear the torsional natural frequencies and the harmonic response to gear excitation are calculated. In another example of a compressor train with a Vorecon the torsional natural frequencies and a short circuit response are calculated. All examples have a practical background from troubleshooting and engineering work although they do not exactly correspond to real cases.

1 Introduction

In industrial machinery gears are widely used. Parallel shaft gears consist of a wheel and one or several pinions. They have limited gear ratios and power rating. More complex arrangements such as planetary gears are used for their high power density and high speed ratio.

The rotordynamic analysis of shaft trains that include complex gears requires the use of appropriate models and tools. In the following the modelling of such trains and their analyses are presented. All the analyses in this paper were carried out with the general rotordynamic program MADYN 2000 [1]. Only the most common analytical features for systems, where gears play an important role, are presented in this paper.

2 Model Types and Analytical Capabilities

2.1 Programs with Lumped-Mass Models

In the past lumped-mass models were widely used for torsional analyses (for example DRESP, see [2]). Even nowadays they are applied. In such models the mass is not continuously distributed, but concentrated at single nodes. It is common to summarise the mass of several sections to one lumped mass. The masses are connected by torsional springs representing the stiffness of the sections between the nodes with masses. This can lead to considerable inaccuracies, especially in the higher frequency range.

In case of a geared system, the model is reduced to a one-speed shaft line which uses the speed of one of the branches as reference. The presentation of the model and shapes then are not very clear. The coupling of lateral and torsional vibrations in gears cannot be considered.

Analytical capabilities are wide and include eigenvalue analyses, harmonic response analyses as well as linear and nonlinear transient response analyses.

Since these programs allow only torsional analyses a separate program for lateral analyses is required.

2.2 General Multi-Body Systems

General multi-body programs offer the possibility to model complex physical behaviours. However they are not focused on rotordynamics and require bigger efforts for modelling. They also do not provide some necessary features such as the calculation of the properties of fluid film bearings for example.

General multi-body software such as SIMPACK [3] are focused on time-domain transient calculations and do not have complex eigenvalue solvers, which are now commonly available in finite element programs.

2.3 MADYN 2000

In MADYN 2000 the rotor structures are modelled with 1-D finite elements according to the Timoshenko beam theory considering the shear deformation and gyroscopic effects. Each cross section can have superimposed cross sections with different mass and stiffness diameter as well as different material properties. The shafts are modelled with their real geometric features, not with lumped-mass substitutes. For section inertias a consistent mass matrix is used.

The shafts can be connected with different types of connectors: Rigid shaft-to-shaft connections, flexible couplings, gear meshes, shaft-in-shaft connections via bearings. For connections to a planet carrier a special connector exists. Complex gears and coupled shaft lines can be modelled with the help of these connectors. The user can set the degrees of freedom of each shaft. Lateral, torsional or axial analyses can be carried out, or a combination of any of these analyses. Radial bearings are readily available and can be modelled with their speed- and load- dependant properties using specialised CFD solver which is integrated in MADYN 2000 [5].

Thus in a lateral-torsional coupled system the contribution of the bearings to the torsional damping can be considered. Moreover the coupled analysis allows to simulate lateral gear vibrations caused by torsional excitation, and vice versa to calculate torsional response to lateral excitation such as gear unbalance.

Each shaft has its own speed. The model is not reduced to a single reference speed. The program checks the shaft speeds and directions of rotation on the basis of the system connections. Shafts connected by a flexible coupling should have the same speed. Shafts connected by gear meshes should have speeds according to a certain speed ratio. These automatic checks are very helpful for complex systems.

The program offers the possibility to carry out static analyses, complex eigenvalue analyses, harmonic response analyses as well as linear and non-linear transient response analyses. The model and the results are stored in one file and all analysis steps (model, calculation and post-processing) are controlled from one interface. Consistency of results and models is automatically ensured.

2.3.1 Basic gear model

A basic parallel-shafts gear is modelled with two shafts (a gear wheel and a pinion) and a "gear connection". It is moreover possible to connect several pinions to one gear wheel or to use several gear connections between two shafts (for example to model a double-helical gear). It is also possible for a shaft to be part of one gear through a gear connection, and be part of another gear through another gear connection. The flexibility that results from the use of such a gear connection allows the modelling of complex gears such as multi stage planetary gears for example.

The gear connection consists of two rigid elements from the shaft centres of the pinions and the wheel to the mesh. The two rigid elements are connected with a directional gear spring at the mesh location. Thus the pitch radii of the wheel and pinion (the gear ratio is the ratio of the pitch radii), the contact angles, the teeth stiffness in direction of the contact angles, the kind of meshing (inner or outer mesh) and the relative angular position of the pinion with respect to the wheel are modelled. Examples of basic gear connections are shown in figure 1. The model of a real double helical gear is shown in figure 2.

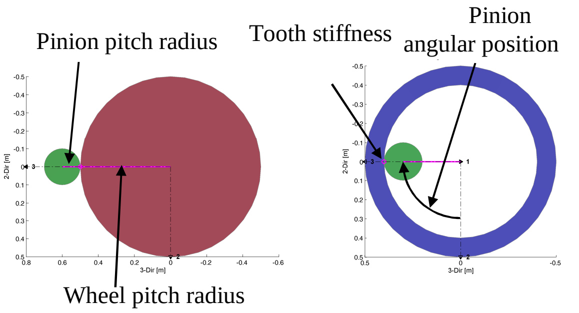


Figure 1: Examples of gear connections: outer mesh (left) and inner mesh (right).

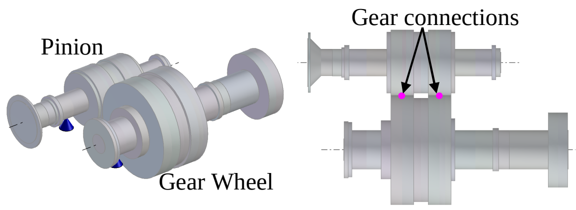


Figure 2: Double helical gear with two gear connections.

2.3.2 Planetary gear model

A planetary gear is modelled with two gear connections per planet: One gear connection with outer mesh that connects the sun to the planet and one gear connection with inner mesh that connects the planet to the annulus. This layout is shown in figure 3.

It is possible to have several planetary gears in one system. The planetary gears can be coupled, e.g. the planet carrier of a planetary gear can be the annulus of another planetary gear. Such a design allows for high gear ratios in a compact layout.

Planets with a rotating axis have a rotating planet carrier. The planet carrier therefore has to be modelled as a rotor and connected to the planets or the planet axes, respectively.

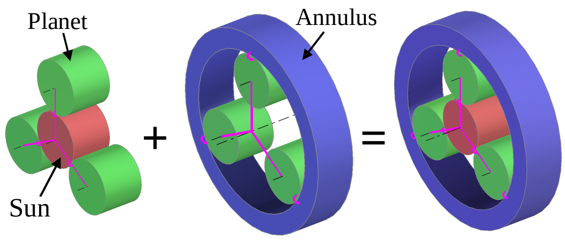


Figure 3: Basic model of a planetary gear (planet carrier not shown).

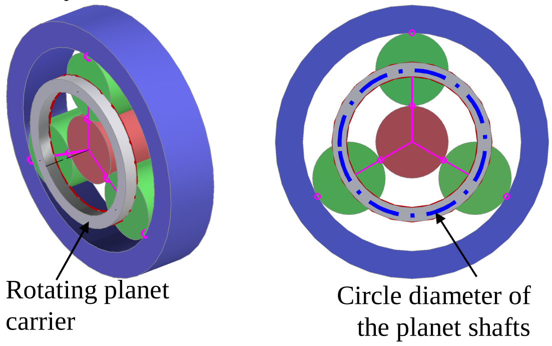


Figure 4: Simplified torsional model of a planetary gear with a rotating planet carrier.

The main property of this connection is the circle diameter of the planet shafts (see figure 4). When a rotating planet carrier is defined the planets must be free in the lateral directions, since rotation of the carrier causes a lateral movement of the offset axes. This is not the case for a pure torsional analysis with a stationary carrier.

For a pure torsional analysis it is not necessary to model the bearings. For a lateral-torsional coupled analysis, however, it is possible to model the planet bearings and supports.

2.3.3 Automatic correctness check of speeds

Many special equations for shaft speeds are known for particular types of gear systems, but a single universal method is needed for use in the software. Although all following equations may seem obvious to every engineer, authors haven't found them published in such general mathematical form.

The main idea of the method is to build several linear equations for each connection point and solve the entire system of equations considering the user inputs as boundary conditions. The specialty of the approach is the consideration of lateral velocities of shaft axes during the calculation as they occur for planet axes with a rotating carrier:

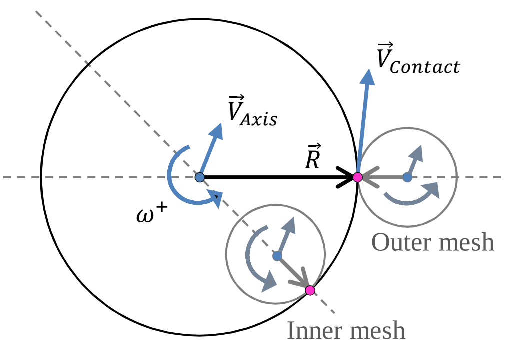


Figure 5: Diagram of the coordinate system

The direction of rotation is considered in the sign of ω, therefore the value is negative for the "Outer mesh" shaft in figure 5. The instantaneous velocity at the contact point of each shaft can be calculated as follows:

$$\vec{V}_{Contact} = \vec{V}_{Axis} + \vec{R} \cdot \omega \tag{1}$$

The direction of the vector $\vec{R}$ depends on the position of the respective shaft. In particular, if the shaft is connected to the outer mesh of another shaft, vectors are opposite.

The velocities at the contact point for a pair of shafts must be equal:

$$\vec{V}_{Axis\,1} + \overrightarrow{R_1} \cdot \omega_1 = \vec{V}_{Axis\,2} + \overrightarrow{R_2} \cdot \omega_2 \tag{2}$$

The radius-vectors are given by the model, but both $\vec{V}_{Axis}$ and ω are considered unknown variables for all shafts at this moment. For linear rotor dynamics with small perturbations of a stationary condition these velocities represent linearized small deviations. Therefore the vectors R or V_{Axis} do not change in time with the rotation of a planet carrier.

In the frequent case of stationary shaft axes, as in a parallel gear, this equation transforms into the well-known form:

$$\omega_1/\omega_2 = -R_2/R_1 \tag{3}$$

This relation alone is insufficient for models with a rotating planet carrier. The connection of a Planet carrier to a pin (see for example inset on Fig. 13) is a special case of equation (2), where $R_2 = 0$, because the connection acts at the axis of the pin shaft with no offset:

$$\vec{V}_{Axis\,1} + \overrightarrow{R_1} \cdot \omega_1 = \vec{V}_{Axis\,2} \tag{4}$$

The pin shaft is rigidly attached to its carrier; therefore their rotational speeds must be equal in the global coordinate system:

$$\omega_{Pin} = \omega_{Carrier} \tag{5}$$

In simplified models for pure torsional analyses (where all bearings are ignored) planet shafts might be directly mounted onto the carrier. The pin and equation (5) are not needed then (see fig. 4).

Two kinds of boundary conditions can be set to reduce the unknowns of the system of equations (2) and solve it:

a. Some shafts are stationary

Shafts which are connected to the common stator with a stationary bearing are given zero velocity of their axes, as well as all shafts with no lateral degrees of freedom. Otherwise their $\vec{V}_{Axis}$ remain free.

$$\vec{V}_{Axis\,i} \coloneqq 0 \quad for \quad i \in stationary\ shafts \tag{6}$$

If there are no stationary shafts, then the system is not statically defined and no unique solution exists.

b. Speed of rotation ω is given

The user has to enter rotational speeds for all shafts in MADYN 2000. Nevertheless, this input might be imprecise or partially incorrect.

The proposed algorithm tries to account for as many user-defined shaft speeds as possible going through all shafts in the system:

1. Select the most recently modified shaft among the yet unvisited shafts.
2. Extend the system of equations with the assumption of rotation speed of the selected shaft:

$$\omega_i \coloneqq \omega_i^{user-defined} \tag{7}$$

3. Solve the system for all unknowns: ω and $\vec{V}_{Axis}$
4. In case of a conflict with previously accepted value of any of the unknowns, reject the assumption (7) and skip steps 5 and 6.
5. Mark shafts, where ω matches user input within given tolerance, as accepted and visited.
6. Save the solution as current best guess.
7. Repeat the algorithm for remaining shafts.

The solution may either confirm consistent user input or suggest a correction with a diagnostic message.

Tables 1 and 2 illustrate this algorithm for the model shown on fig. 6 of a motor, gear (with ratio 1:4.367) and a compressor. For demonstration purposes it is assumed that incorrect speed is entered for the Pinion, and that a fluid coupling is installed between pinion and compressor. Therefore speeds of the connected shafts are not directly related – there are two independent sub-systems.

All shaft axes are stationary; therefore all $\vec{V}_{Axis}$ are zero and aren't shown in the table.

On the first iteration, equation (7) is added for the motor shaft, setting 2000 rpm (shown underlined) as a boundary condition. The resulting solution of the system of equations already matches user input for shafts 1 and 2 (shown in bold), so nothing has to be done for the wheel.

Shaft	Iteration 1 ω Input	Iteration 1 ω Sol.	Iteration 2 Input	Iteration 2 Sol.
1. Motor	**2000**	**2000**	**2000**	**2000**
2. Wheel	2000	**2000**	2000	**2000**
3. Compr.	-6000	*0*	**-6000**	**-6000**
4. Pinion	-8000	*-8734*	-8000	*-8734*

Table 1: Solutions of the system on iterations 1–2.

Shaft	(Iteration 3) ω Input	(Iteration 3) ω Sol.	Result
1. Motor	**2000**	*1992*	2000
2. Wheel	2000	*1985*	2000
3. Compr.	**-6000**	-6000	-6000
4. Pinion	**-8000**	*-8001*	**-8734**

Table 2: Rejected approximate solution of the system on iteration 3 and final solution.

Result for shaft 3 does not match, because its speed is independent from the rest of the system; for shaft 4, because of wrong user input. The algorithm does not make a distinction between these two cases.

On the second iteration, another boundary condition is added for the Compressor shaft (number 3). The solution is now correct, but not all user inputs have been verified, so the algorithm continues.

On the third iteration, program tries to validate the solution for the Pinion by enforcing its given speed as yet another boundary condition. This leads to over-defined system of equations and an approximate solution. This newly introduced constraint is rejected, because it does not hold in the solution and previously accepted results for shafts 1 and 2 became wrong as well. This contradiction means that given speed of the Pinion is wrong.

Finally the program accepts the result of the second iteration and concludes that pinion must be rotating at 8734 rpm in the negative direction using the given motor speed as a decision basis.

If not all shaft axes in the system are fixed as in the example described above, each unknown vector $\vec{V}_{Axis}$ is numerically represented as two scalar projections onto the global coordinate system used in MADYN 2000. A couple of such systems are described in details in the following section:

Example 2 (Mill Train, fig. 13) contains 18 shafts and 25 connections of different kinds. This leads to a system of 56 equations and 54 scalar variables.

Example 3 (Vorecon, fig. 18) consists of two speed-independent subsystems. It contains 20 shafts and 27 connections. Resulting system of equations has 60 unknowns, constrained by 96 equations. All but 4 shafts are stationary (boundary condition of type ‘a’), so the system can be significantly reduced to just 28 unknowns (20 shaft speeds + 2 projections of $\vec{V}_{Axis}$ of 4 revolving planets) and 31 equations. The complete solution is fully defined in just two iterations. As in the Motor-Gear-Compressor example only 2 boundary conditions of type ‘b’ are needed.

3 Examples of Analyses

3.1 Coupled Lateral-Torsional Analysis of a Geared Compressor Shaft Train

The vibration behaviour of a compression train is analysed, with particular attention to the lateral vibration behaviour of the gear pinion and its contribution to the damping of torsional modes. The system can be seen in figure 6.

Forces at the gear shaft bearings (hence the bearing characteristics) strongly depend on the torque in the geared shaft train. Therefore the dynamic behaviour of the gear shafts must be analysed for all pertinent load cases. Here the analysis focuses on low gear loads (10% of the nominal load).

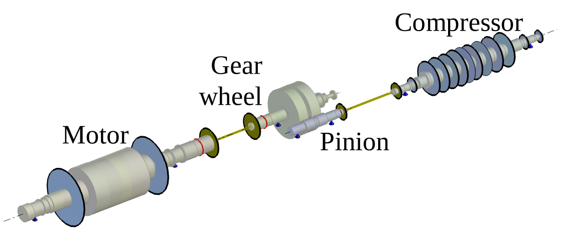


Figure 6: Torsional model of the compressor train.

The centrifugal compressor is driven by an electric motor through a parallel-shaft gear. All machines are connected with flexible couplings.

As a first step independent torsional and lateral analyses are carried out. The torsional analysis is done with a pure torsional model of the train, i.e. with a model, where the only degree of freedom is angular displacement around the axis of rotation.

The first three torsional modes of the train are shown in figure 7. The shaded red curves represent the torsional angle scaled with the gear radii.

For the lateral analysis of the pinion a stand-alone lateral model is used. The bearing coefficients of the pinion are calculated within MADYN 2000. After determining the static bearing forces for the considered gear load, the stiffness and damping coefficients are calculated as a function of the speed (with speed-dependent temperature and viscosity).

Two types of bearings are considered: 2-lobe bearings and 4-lobe bearings. The first three lateral modes of the pinion for operation at nominal speed and 10% of the nominal load are shown in figure 8 and 9. With both types of bearings the modes are well damped.

From these results no problems are expected, since the torsional modes are not in resonance with relevant excitations and the lateral modes in the speed range of the pinion up to 7’800 rpm are well damped.

In addition to these standard analyses a coupled lateral-torsional analysis is carried out. In the model the gear shafts, in which coupling between the lateral and torsional displacement can occur, have their lateral and torsional degrees of freedom free. The motor and the compressor only have their torsional degrees of freedom free. The coupled modes of the train are shown in figures 10 (2-lobe bearings) and in figure 11 (4-lobe bearings). The operating speed is the nominal speed; the gear load is 10% of the nominal load. It can be seen that the torsional modes T1 and T2 combine rotational displacement in all shafts (in red), and lateral displacement in the pinion (in blue). With the 2-lobe bearings the damping ratio of the 2nd torsional mode is negative, i.e. the mode is unstable.

The lateral tilting modes of the pinion also change in the coupled analysis.

In [4] a similar case is reported for a compressor train, which exhibited very high non-synchronous pinion vibration after start-up, when the load in the compressor was still low. The problem was resolved by replacing the 2-lobe bearings of the pinion by 4-lobe bearings.

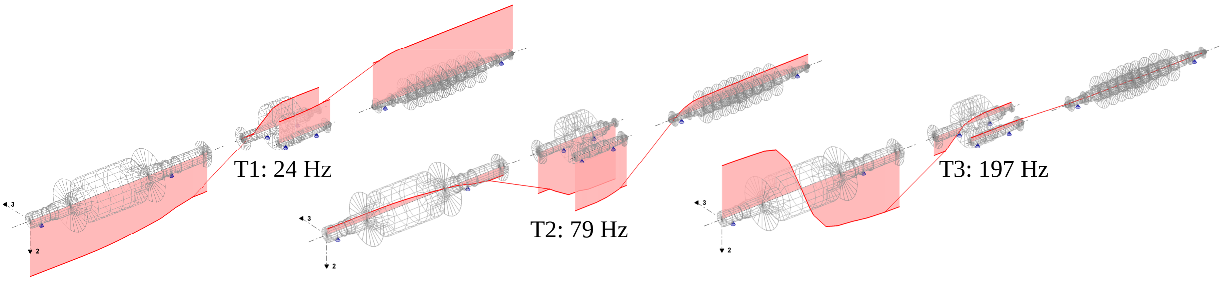


Figure 7: Train torsional modes.

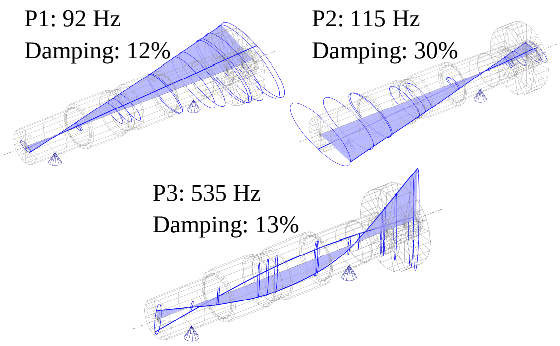


Figure 8: Pinion modes, 2-lobe bearings, 10% load.

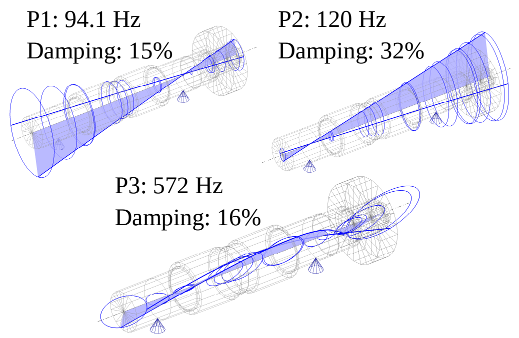


Figure 9: Pinion modes, 4-lobe bearings, 10% load.

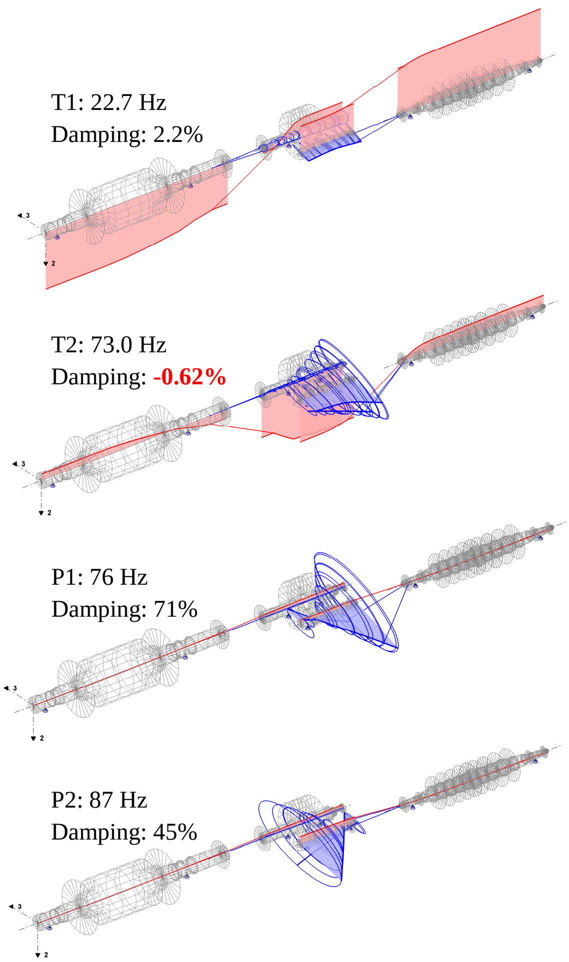


Figure 10: Coupled modes, 2-lobe bearing, 10% load.

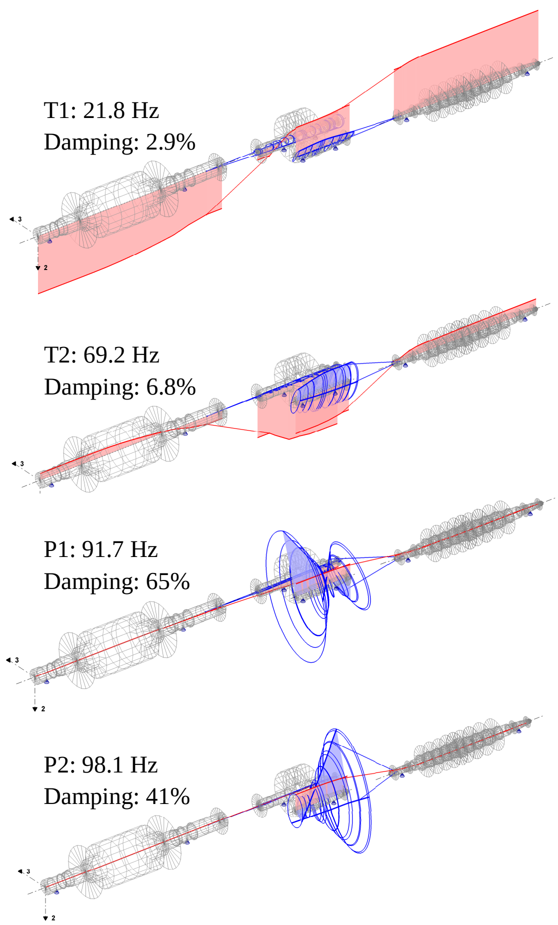


Figure 11: Coupled modes, 4-lobe bearing, 10% load.

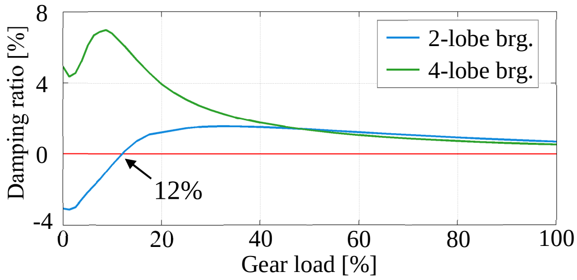


Figure 12: Damping of mode T2 versus power.

In figure 12 one can see the damping ratio of the 2nd torsional mode as a function of the power transmitted by the gear. It is shown that for a power below 12% of the nominal power the 2nd torsional mode is unstable with the 2-lobe bearings. With the 4- lobe bearings the damping ratio of the mode is always positive.

3.2 Torsional Natural Frequencies of a Mill Train

The shaft train of this example consists of a fixed speed electric motor, a two-stage planetary gear used in a mill. The overall speed ratio is 40. The motor is connected to the gear through a rubber coupling and a 90° bevel gear which provides a first stage of speed reduction. In the gear itself there are two planetary gears. The sun of the first stage is connected to the bevel gear output shaft through a toothed coupling. The annulus of this first stage is rigidly connected to the sun of the second stage, while the planet carrier of the first stage is connected to the annulus of the second stage which is also the output shaft. Hence the planet carrier of the first stage is rotating, whereas the planet carrier of the second stage is fixed. A plot of the train model is shown in figure 13.

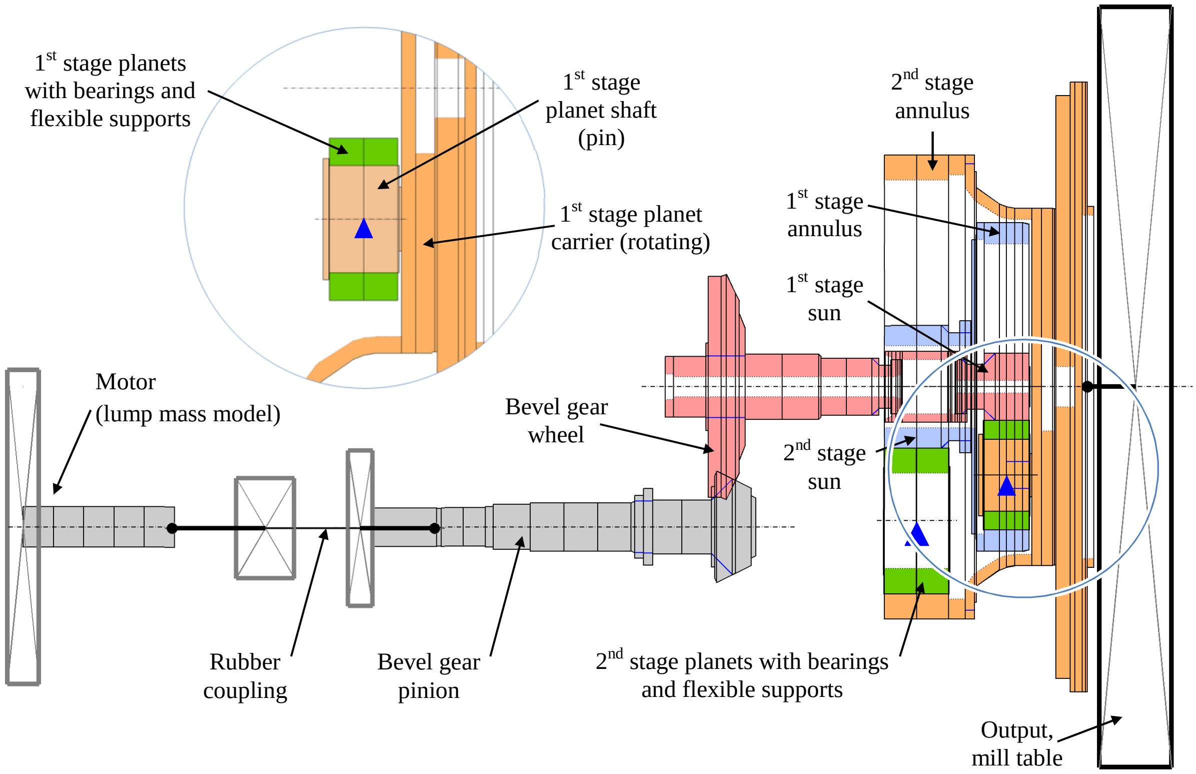


Figure 13: Model of the mill train (motor and bevel gear shown in the same plane as the gear).

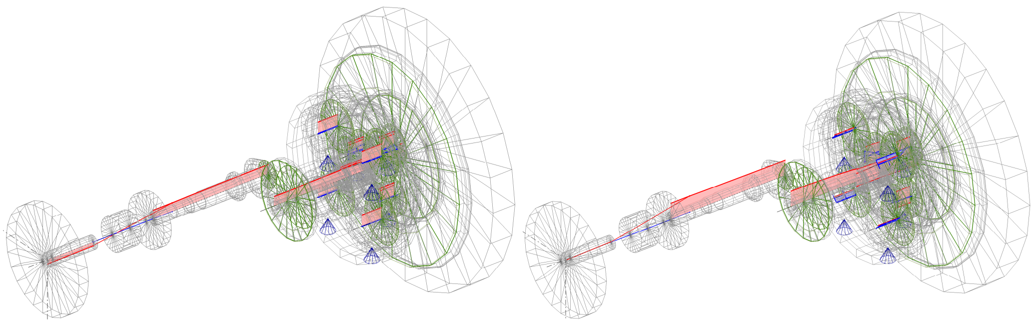
One of the objectives of the analysis is to calculate the torsional natural frequencies of the complete train and verify that none of them was in a prohibited range. The damping in the rubber coupling is taken into account. Moreover the influence of the flexibility of the planet shafts and of the planet bearings on the modes shall be checked. Therefore a coupled lateral-torsional system is required.

The loads are applied as torques and the bearing loads are automatically received.

Throughout the modelling process the system is continuously checked with the automatic correctness checks provided by MADYN 2000. Before proceeding with the eigenvalue analysis a further check is carried out with a static analysis. A rotational angle is applied at the input shaft and it is checked if the output angle is according to the gear ratio.

The mode shapes of the 1st and 2nd mode are shown in figure 14. There are no modes in the forbidden frequency window.

The 1st mode has its main deflection in the rubber coupling. Its damping mostly comes from the coupling and to some extent from the bearings.



1st torsional mode 7.35 Hz
coupling damping 6.2%, bearing damping 0.4%

2nd torsional mode 27.0 Hz
coupling damping 1.3%, bearing damping 0.6%

Figure 14: 1st and 2nd torsional modes.

The lateral-torsional coupling is stronger for the 2nd mode, as can be seen from its higher bearing damping. In figure 15 the displacement in the bearing and in the planet shaft of the 1st stage planet is shown for this mode.

The natural frequency of the 1st mode is close to the speed of the shaft line consisting of the bevel gear wheel, the toothed coupling and the sun of stage 1. The sensitivity of the 1st mode to an excitation in the sun of the 1st stage was analysed with a harmonic response analysis. A total damping ratio of 7.2% was considered: 6.2% from coupling damping, 0.4% from bearing damping and 0.625% from structural damping.

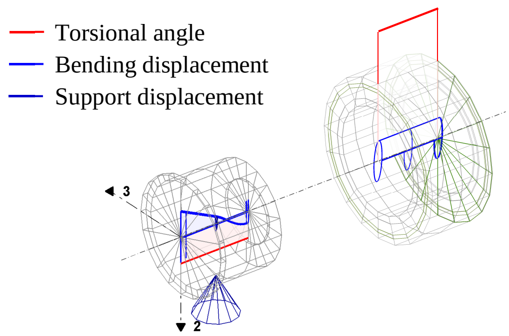


Figure 15: 2nd torsional mode shape, 1st stage planet and 1st stage planet shaft.

An excitation amplitude of 1% of the rated torque was considered. The results of the harmonic response analysis are shown in figure 16.

The response in the rubber coupling is 2’670 Nm, which is a few percent of the nominal motor torque. The response forces in the five gear meshes are also just a few percent of the nominal tangential gear mesh forces. The maximum shaft stress occurs in the bevel gear wheel shaft (1.1 MPa).

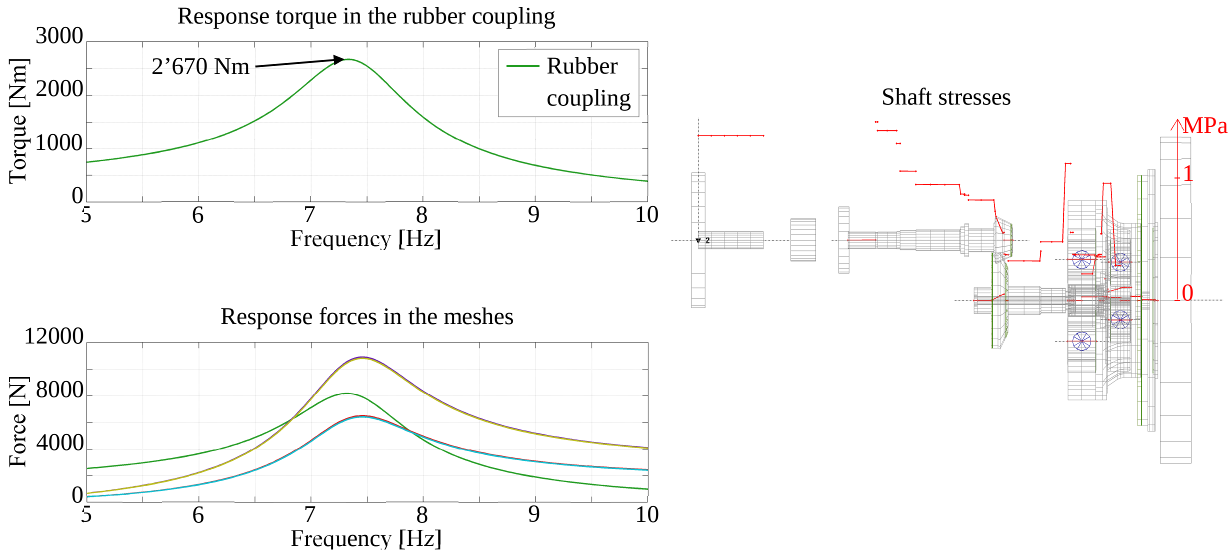


Figure 16: Results of the harmonic response analysis.

3.3 Torsional Analysis of a Compressor Shaft Train with a Vorecon

In this example the shaft train includes a fixed speed synchronous motor, a Vorecon and a centrifugal compressor. The system is shown in figure 17 and the Vorecon in figure 18. The motor is connected to the primary shaft of the Vorecon through a rubber coupling. The coupling between the Vorecon and the compressor is a membrane coupling.

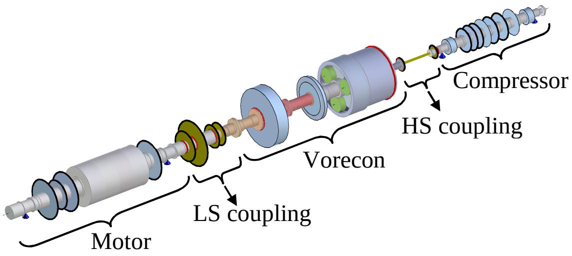


Figure 17: Torsional model of the compressor train.

The variable speed in the high speed section is obtained by means of the Vorecon. The Vorecon is a gear with a fixed input speed and a variable output speed. The output gear stage is a planetary gear with a rotating planet carrier. Speed variation is obtained by varying the speed of the planet carrier. The planet carrier is driven by a hydraulic torque converter that takes its power from the input shaft. The output speed of the torque converter is varied by regulating the oil flow that goes through it. Its speed is reduced by a planetary gear with a fixed planet carrier to drive the planet carrier of the output planetary gear.

The purpose of the analyses is to determine the torsional natural modes in operation and the transient responses to run-up and short circuit excitations.

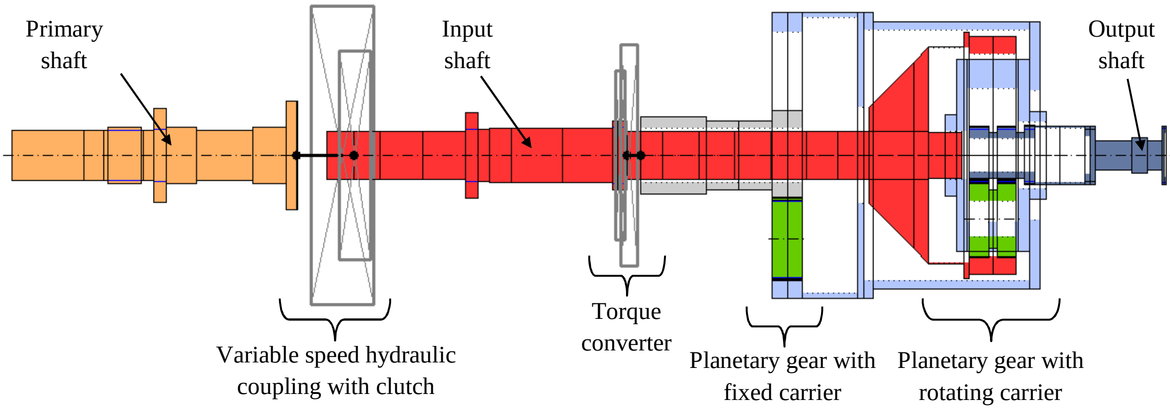


Figure 18: Model of the Vorecon.

The torsional mode shapes of the first four modes of the train are shown in figure 19. The first mode (16.04 Hz) has the motor vibrating against the rest of the train. The deformation occurs in the low speed coupling and in the input shaft of the Vorecon. The second mode is the mode of the variable speed hydraulic coupling. For both modes the displacement in the input shaft is shown in figure 20.

The transient analyses were carried out with consideration of the non-linear behaviour of the rubber blocks in the low speed coupling and of the individual mode damping ratios. The response torques in the couplings in case of 2-phase short-circuit can be found in figure 21. All torques are plotted in p.u. (= Power Unit) and refer to the motor power and nominal shaft speed. The torques are below the maximal allowable torques of the couplings.

4 Summary

The modelling and rotordynamic analysis of complex gears is illustrated with three examples: A parallel-shaft gear, a multi-stage planetary gear and a Vorecon. The influence of the bearings, the bearing supports and the coupling between lateral and torsional vibrations are considered. These extended analytical capabilities provide engineers a tools for the design of modern machinery and the simulation of complex phenomena in case of troubleshooting.

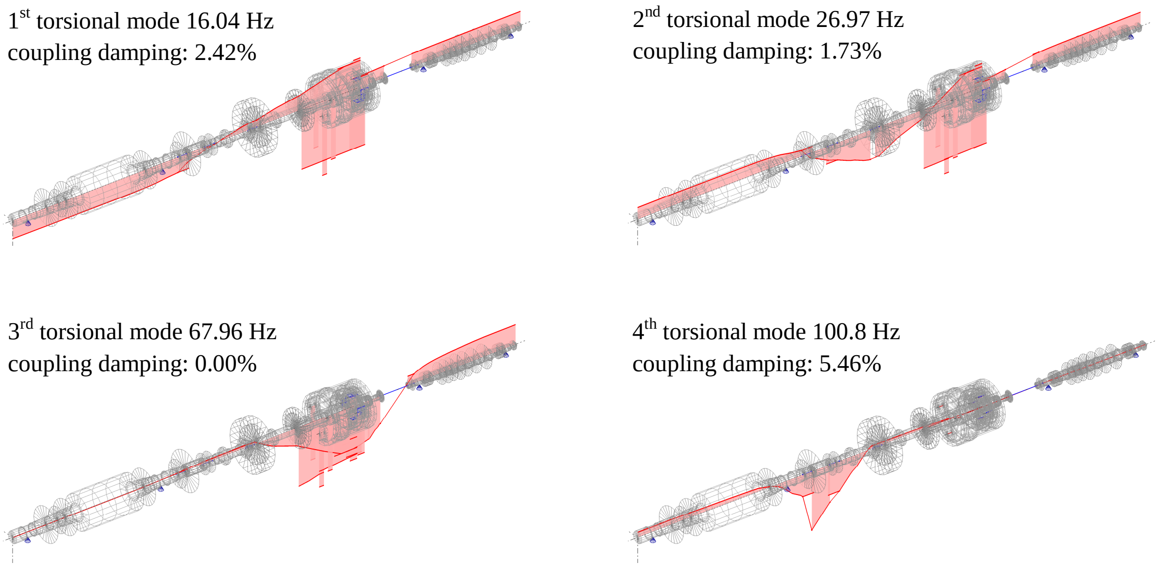


Figure 19: Torsional mode shapes of the train

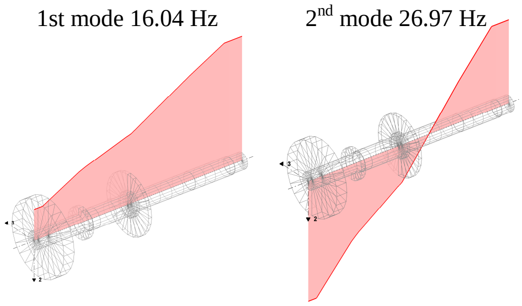


Figure 20: 1st and 2nd torsional mode shapes, Vorecon input shaft.

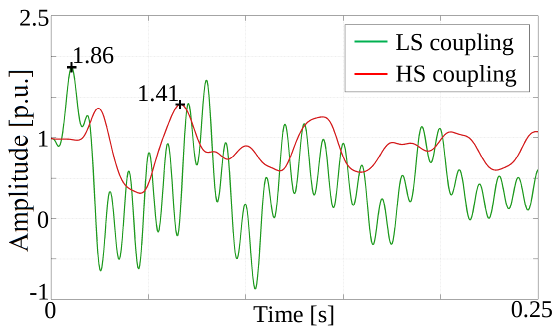


Figure 21: Transient 2-phase short circuit response, coupling torques.

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